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AN APPLICATION OF A COLORED NOISE KALMAN FILTER TO A RADIO-GUIDED ASCENT MISSION

Prepared by

D. J. Johnson
Guidance Analysis Department
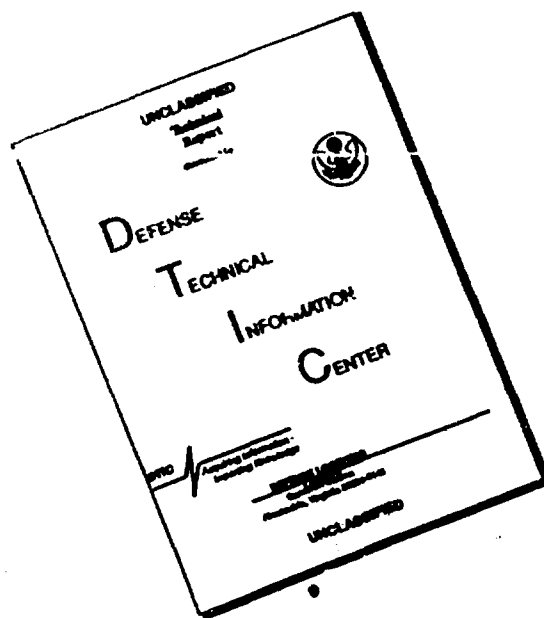Electronics Division

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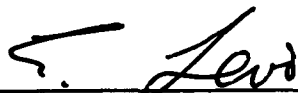Los Angeles, California

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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California under Air Force Contract No. F04701-69-C-0066. This report, which documents research carried out from August 1968 through March 1969, was submitted on 18 August 1969, for review and approval, to Richard A, Binger, 2nd Lt.. USAF.

Approved

E. Levin, Director
Guidance and Navigation Subdivision
Electronics Division
Engineering Science Operations

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Richard A. Binger, 2nd Lt., USAF
Project Officer

Abstract

A set of general sequential filter equations is
derived for nonlinear system dynamics and a non-
linear observation model, but is obtained with the
assumption of a linear estimator. These equa-
tions, which are based upon previously developed
formulas, include the colored measurement noise
statistics and the statistics of nonestimated model
parameter errors. Several simulations made with
this filter are compared with the simulation
results of an estimation procedure constructed
with the utilization of the standard white noise
assumptions. The difference between the white
noise filter results and the colored noise filter
results is found to be minimal. Instability occur-
red when the statistics of the effective exhaust
velocity in the acceleration model were not proper-
ly accounted for.

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CONTENTS

FIGURES

I. Introduction

During radio-guided launch missions, the angle measurement noise of the guidance radar becomes increasingly bothersome as the radar line-of-sight tends toward low elevation angles. Significantly, a greater portion of the launch guidance has been occurring at low elevation angles as a consequence of the more intensive use of larger boost vehicles, which have become available to place heavier payloads in orbit. The bothersome component of the angle measurement noise is due to random fluctuations in the tropospheric index of refraction. These fluctuations are most noticeable in the denser portion of the atmosphere and are directly attributable to the random nature of temperature, pressure, and humidity variations. These variations are slower and more severe in the low-altitude regions and thus affect not only the magnitude but also the autocorrelation of the angle errors when the radar energy traverses these regions. In fact, angle noise correlation times of several seconds are common at radar elevation angles of a few degrees. In addition, the present trend in guidance filtering has been toward more intensive use of Kalman filtering techniques. Since filters of this type require that the measurement noise characteristics be modeled, it is appropriate to investigate the behavior of such filters when various assumptions are made concerning the noise correlations.

Error sources other than measurement noise affect the guidance filter behavior and are of prime concern in a radio-guided launch mission. A filter constructed to minimize the effects of measurement noise will itself introduce errors into the estimate that arise from inaccuracies in specifying the mathematical model of the system. These modeling errors reflect the limits in human ability to describe the real world in exact mathematical terms. Errors of this sort can have an accumulative effect over a period of time and can cause the estimate to diverge. It is assumed for the purposes of this paper that the mathematical form of the system model is known or can be adequately represented by some series expansion; however, the values of certain parameters of the model are in error. This assumption differs significantly from the situation where the mathematical form itself is uncertain. Examples of model parameter uncertainties are survey uncertainties, measurement bias errors, and parameter errors such as the uncertainty in the numerical value of the gravitational constant.

In this paper a derivation is given of a general sequential filter which utilizes the differencing scheme of Refs. 1 and 2 to account for sequentially correlated noise. Nonestimated model parameter uncertainties in both the state dynamic model and the measurement model are accounted for in the derivation.

The various features of the general filter were programmed in a simulation of a launch guidance mission and the results are given here. The primary purpose of the simulation was to examine the behavior of the general filter and to determine whether its performance was such as to warrant an increase in the complexity of existing guidance programs.

II. The Optimum Filter

Before proceeding into the mathematical details of the derivation, a qualitative description of the filter will be given to help in visualizing the filtering process in the presence of correlated measurement noise and model parameter errors. An optimum linear sequential filter operates on data samples that are available at discrete values of time. Its output at a particular time is an estimate of the system state based on all information available up to that time. A block diagram of a sequential filter is shown in Fig. 1. It is optimum in a least squares or minimum variance sense, providing that the Gaussian and linear propagation properties of the errors hold; and it is linear because the estimates are formed from a linear combination of the observation data.
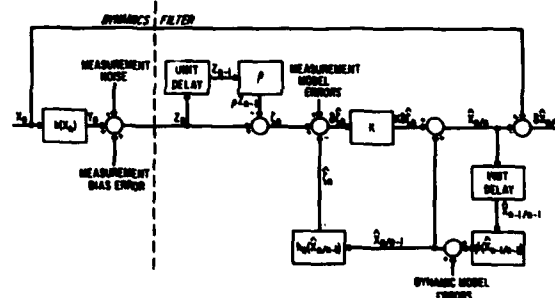

Fig. 1. Sequential Filter Block Diagram

At the beginning of each computation cycle there are available an estimate of the state vector and a covariance matrix of errors in the estimate. Also input at this point are a set of measurements and a covariance matrix of measurement errors. These data -- the state estimate and the measurement set -- are weighted according to the relative

magnitude of their respective covariances, data with larger covariances being weighted less than those with smaller covariances, and are appropriately transformed to form a new state vector estimate. Next, a new state vector estimate error covariance matrix is computed by propagating the previous state vector estimate error covariance matrix and the measurement error covariance matrix through the filtering equations. Finally, the state vector estimate and the state vector estimate error covariance matrix are propagated to the beginning of the next computation cycle by the filter dynamic and error propagation models.

Consider now the situation where the measurement noise is highly correlated, as would be the case in low-elevation radar tracking. The estimate errors will be correlated with the measurement noise through previous measurements. A filter based on white noise assumptions will ignore this fact and improperly propagate the covariance matrices, consequently causing improper weighting of the data. For example, suppose that the range from a radar to a fixed point were to be determined from range measurement samples and that these measurements possessed noise with long-term drift characteristics. A white noise filter would treat the measurement noise as being statistically independent from sample to sample with a statistical mean that tended toward zero as the number of samples increased, when in actuality the measurement error would behave more as a bias if the total time span under consideration is short.

One means of accounting for the presence of colored noise is to augment the state vector to include the states of a hypothetical shaping filter whose input is white noise and whose output is noise of the required correlations. This approach is often prohibitive primarily because of computer storage limitations, since for each increase in the state vector dimensions there is a corresponding increase in the dimensions of the state covariance matrices and in the complexity of the ensuing matrix arithmetic. Also, the state augmentation scheme can result in ill-conditioned matrices[1]. A second method which utilizes a measurement differencing technique[1],[2] reduces the implementational difficulties associated with state augmentation and at the same time alleviate the threat of matrix inversion problems. Because of these advantages, the second method was used in the launch guidance simulation. Recently, a third method was developed which does not depend upon state augmentation or measurement differencing. It was, however, developed too late to obtain results for the paper, and hence will be discussed in a subsequent report[3].

Basically, there are three ways in which the model parameter errors can be accounted for in the estimation process. First, they can be ignored completely. In this case the state vector estimate error covariance matrix, since it will not show the effect of propagating the parameter errors, will be smaller than it should be. Consequently, the state vector estimate that is present at the beginning of each computation cycle will be weighted more and the measurements weighted less than if the effects of the parameter errors were accounted for.

Secondly, the parameters can be estimated along with the rest of the state vector elements in hopes of driving the bothersome errors to smaller

values. In any event, the parameter error covariances will be included in the state vector estimate error covariance matrix and will hence influence the computation of the filter weights. That is, even if the parameter uncertainties are not reduced, they will be accounted for in the filter error analysis.

A third way to account for the model parameter uncertainties is similar in some respects to the second method. In this method the error covariances of the parameter errors are included in the filter covariance matrix propagations[4], but unlike the second method the parameters are not solved for, and thus the parameter errors never decrease. The procedure is equivalent to the situation where the parameter is solved for as in the second method but where poor parameter estimates are obtained. Attempts have been made to approximate this third approach by the addition of an artificial "state noise" covariance matrix to the state error covariance matrix once during each computation cycle. Although such an approach is not a systematic one, it is obviously better than ignoring the parameter errors as the first method does since it prevents the state error covariance matrix from becoming too small. Nevertheless, the correlation between the state estimate error and the parameter error is ignored inasmuch as the state noise approach is based on the assumption of white noise parameter errors.

III. Derivation of the General Filter

Let the state dynamics be described by

$$X_n = \varphi(X_{n-1}, p, \mu_n) \qquad (1)$$

and the relation of the measurement to the states by

$$z_n = h(X_n, T) + N_n \qquad (2)$$

Let the measurement noise be generated by the Markov process

$$N_n = \rho N_{n-1} + q W_n \qquad (3)$$

The filter dynamic model is assumed to be of the form

$$\hat{X}_{n/n-1} = \varphi(\hat{X}_{n-1/n-1}, \hat{p}) \qquad (4)$$

and the filter measurement model of the form

$$\hat{z}_n = h(\hat{X}_{n/n-1}, \hat{T}) \qquad (5)$$

Let the measurements be differenced to give an effective measurement

$$\zeta_n = z_n - \rho z_{n-1} \qquad (6)$$

The measurement estimate is

$$\hat{\zeta}_n = h(\hat{X}_{n/n-1}, \hat{T}) - \rho h(\hat{X}_{n-1/n-1}, \hat{T})$$

$$= h_a(\hat{X}_{n-1/n-1}, \hat{T}) \qquad (7)$$

Assume that the state vector estimates are obtained from a linear filter of the form

$$\hat{X}_{n/n} = \hat{X}_{n/n-1} + K_n[\zeta_n - \hat{\zeta}_n] \quad (8)$$

The error in the estimate is

$$\delta\hat{X}_{n/n} = X_n - \hat{X}_{n/n} = \delta\hat{X}_{n/n-1} - K_n\delta\hat{\zeta}_n \quad (9)$$

where

$$\delta\hat{X}_{n/n-1} = X_n - \hat{X}_{n/n-1} \quad (10)$$

$$\delta\hat{\zeta}_n = \zeta_n - \hat{\zeta}_n \quad (11)$$

The filter described by the above equations is shown in the block diagram of Fig. 1.

By the Wiener-Hopf Equation (see Appendix) the filter gain is obtained as

$$K_n = E[\delta\hat{X}_{n/n-1}\,\delta\zeta_n^T]\left|E[\delta\hat{\zeta}_n\,\delta\hat{\zeta}_n^T]\right|^{-1} \quad (12)$$

To obtain an explicit expression for K_n it is first necessary to expand Eq. (10). Thus

$$\delta\hat{X}_{n/n-1} = \varphi(X_{n-1}, p, \mu_n) - \varphi(\hat{X}_{n-1/n-1}, \hat{p}) \quad (13)$$

which, if one assumes that the estimate is within a linear expansion of the state, becomes

$$\delta\hat{X}_{n/n-1} = \Phi\delta\hat{X}_{n-1/n-1} + D\delta\hat{p} + \Gamma\mu_n \quad (14)$$

Similarly, Eq. (11) can be written as

$$\delta\hat{\zeta}_n = (H_n\Phi - \rho H_{n-1})\delta\hat{X}_{n-1/n-1} + H_nD\delta\hat{p}$$

$$+ A\delta\hat{T} + H_n\Gamma\mu_n + qW_n$$

$$= H_a\delta\hat{X}_{n-1/n-1} + H_nD\delta\hat{p} + A\delta\hat{T} + H_n\Gamma\mu_n + qW_n \quad (15)$$

After one uses Eqs. (14) and (15) in Eq. (12), and performs the indicated expected values, K_n can be determined from

$$E[\delta\hat{X}_{n/n-1}\,\delta\zeta_n^T] = \Phi P_{n-1}H_a^T + \Phi V_{n-1}D^TH_n^T$$

$$+ \Phi L_{n-1}A^T + DV_{n-1}^TH_a^T$$

$$+ DSD^TH_n^T + \Gamma Q\Gamma^T H_n^T \quad (16)$$

and

$$E[\delta\hat{\zeta}_n\,\delta\hat{\zeta}_n^T] = H_aP_{n-1}H_a^T + qRq^T + H_n\Gamma Q\Gamma^TH_n^T$$

$$+ H_nDV_{n-1}^TH_a^T + H_aV_{n-1}D^TH_n^T$$

$$+ H_nDSD^TH_n^T + AL_{n-1}^TH_n^T + H_aL_{n-1}A^T$$

$$+ ABA^T \quad (17)$$

where

$$P_n = (\Phi-K_nH_a)P_{n-1}(\Phi-K_nH_a)^T + K_nqRq^TK_n^T$$

$$+ (I-K_nH_n)\Gamma Q\Gamma^T(I-K_nH_n)^T$$

$$+ (K-K_nH_n)DSD^T(I-K_nH_n)^T$$

$$+ (\Phi-K_nH_a)V_{n-1}D^T(I-K_nH_n)^T$$

$$+ (I-K_nH_n)DV_{n-1}^T(\Phi-K_nH_a)^T$$

$$+ K_nABA^TK_n^T - (\Phi-K_nH_a)L_{n-1}A^TK_n^T$$

$$- K_nAL_{n-1}^T(\Phi-K_nH_a)^T \quad (18)$$

$$L_n = (\Phi-K_nH_a)L_{n-1} - K_nAB \quad (19)$$

$$V_n = (\Phi-K_nH_a)V_{n-1} + (I-K_nH_n)DS \quad (20)$$

Equations (16) through (20) along with Eq. (12) constitute the general sequential filter.

IV. Application of the Filter

In radio-guided ascent missions a tracking radar measures the position and/or velocity of the launch vehicle (see Fig. 2). These measurements are processed by a guidance computer to obtain an estimate of the vehicle state. The computer determines, according to a programmed guidance law, what vehicle steering is necessary to satisfy the mission requirements; and steering commands are transmitted to the vehicle via the guidance radar link. In addition, the radar transmits discrete commands such as engine cutoff as soon as the computer determines the necessity of such events.

A simulation of a typical launch guided mission was constructed, as shown schematically in Fig. 3, in which a vehicle was guided during its last two stages. The purpose of the simulation was to assess the performance of the general sequential filter in a simulated radar noise environment, and in the presence of uncertainties in both
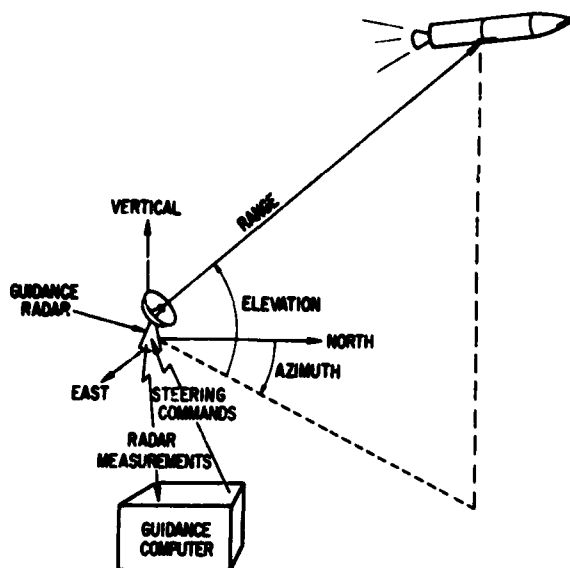
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Fig. 2. Launch Guidance Radar System

Fig. 3. Simulation Block Diagram

the filter state model parameters and filter measurement parameters. The vehicle was assumed to be guided perfectly, hence the state vector in the simulation was independent of the filter estimates. The performance factors were the computed residuals of the state estimation error and the computed state estimation error covariance matrix. A position radar measuring range, azimuth, and elevation was simulated. The simulation error model added random noise to the simulated measurements which possessed the following one-half second correlations:

range noise correlation = 0.1

azimuth noise correlation = 0.95

elevation noise correlation = 0.97

The estimated state was a 12-element vector consisting of the following quantities:

x, y, z - a 3-element ECI Cartesian position vector

$\dot{x}, \dot{y}, \dot{z}$ - a 3-element ECI Cartesian velocity vector

c^* - effective engine exhaust velocity

t_m - engine mass depletion time

i_x - x-component of a roll axis oriented unit vector

i_z - z-component of a roll axis oriented unit vector

Ω_x - pitch axis drift rate

Ω_y - yaw axis drift rate

The following 7 simulations were performed:

a. The filter error model was based on an assumption of white measurement noise (white noise filter). The simulation error model generated measurement noise with the correlations given above.
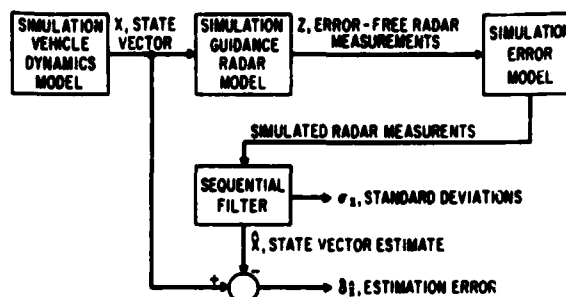
b. The filter error model was based on an assumption of correlated measurement noise with the correlations given above (colored noise filter). The simulation error model generated measurement noise with these same correlations.

c. The filter was a white noise filter, and the simulation error model generated bias errors only.

d. The filter error model was based on an assumption of measurement bias errors, and the simulation error model generated bias errors only.

e. The filter was a white noise filter, the simulated radar measurements were error-free, and c* was omitted from the estimated state vector. No compensation for c* uncertainties was attempted.

f. The filter was a white noise filter and the simulated radar measurements were error-free. c* was included in the estimation vector.

g. This simulation was identical with simulation e except that the c* error statistics were included in the filter equations.

V. Results

Typical results of the first simulation (the white noise filter) are given in Fig. 4a and can be compared with the results of the second simulation (the colored noise filter) given in Fig. 4b. Both figures show the x-components of the velocity estimate residuals. In the same plots the x-velocity standard deviation envelopes obtained from the state estimate error covariance matrix of the filter are also shown. Comparison of the white noise filter velocity residuals of Fig. 4a with the velocity residuals of the colored noise filter in Fig. 4b show that, although there is an improvement in the estimate, the improvement is small, because the average deviation of the residual about the zero mean value is about the same in both simulations. There is, for example, only about a 2 ft/sec difference in the velocity residual at burnout. The filter value of the estimate error standard deviations as shown by the standard deviation envelopes, however, is larger in the colored noise case and is more representative of the estimate errors.
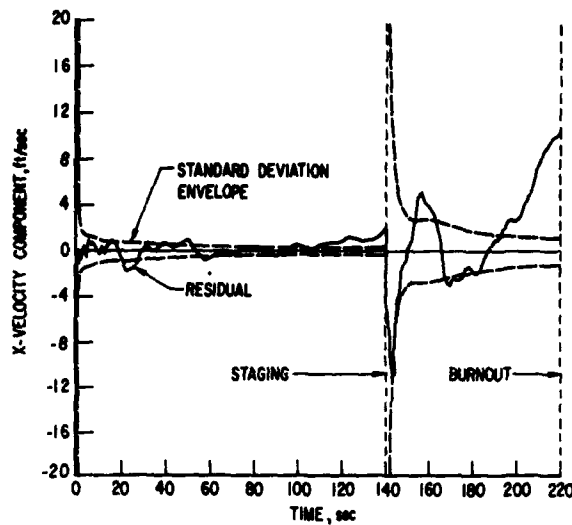
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Fig. 4a. Velocity Residual, White
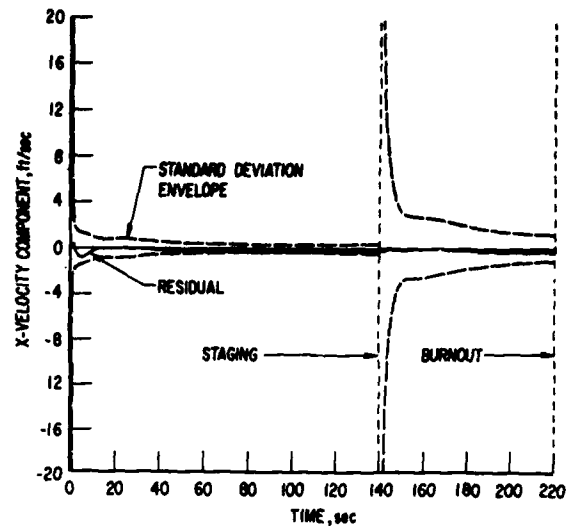Noise Filter

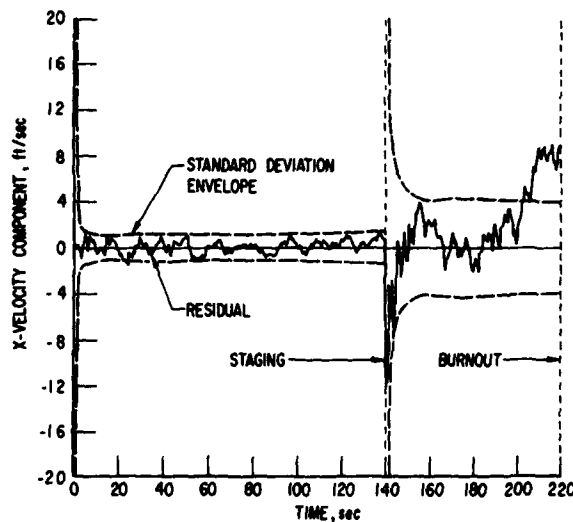Fig. 5a. Velocity Residual, White
Noise Filter

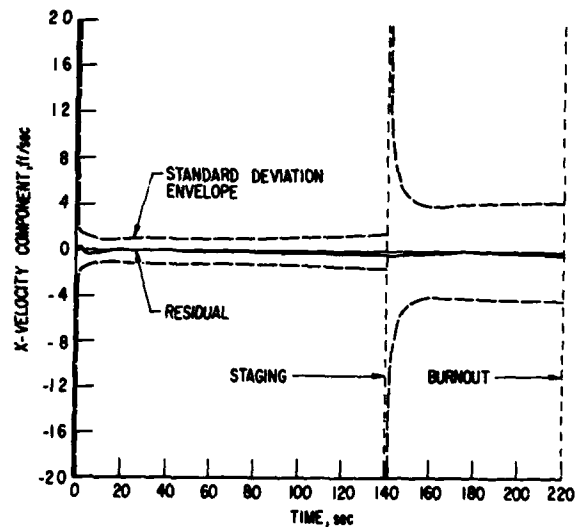Fig. 4b. Velocity Residual, Colored
Noise Filter

Fig. 5b. Velocity Residual,
Bias Filter

The results of simulations c and d showing
the effects of measurement bias errors on the
output of the white noise filter and the bias filter
are given in Figs. 5a and 5b. A small difference
can be seen between the white noise filter results
and the bias filter results. The computed stand-
ard deviations were larger when the bias filter
was used as illustrated by the larger values of the
envelope.

Figures 6 and 7 show the results of the simu-
lation e in which c* was omitted from the state
estimation vector and the c* uncertainties were
ignored. Figure 6 shows that the filter velocity
estimates become unstable after only about 60 sec.
The filter gains plotted in Fig. 7 are the gains
relating the position estimates to the transformed
radar measurements; the radar measurements
are transformed into the same coordinates as the
state vector position components. The gains in

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Fig. 7 become less than 0.4 after the first 10 sec of filtering which implies that the a priori estimate -- the state estimate present at the beginning of each computation cycle -- receives a greater share of the weighting than do the measurements. The somewhat wild behavior of the gains during the second stage is attributed to the unstable behavior of the estimates that are used to evaluate the partial derivatives which, in turn, affect the gain computations.
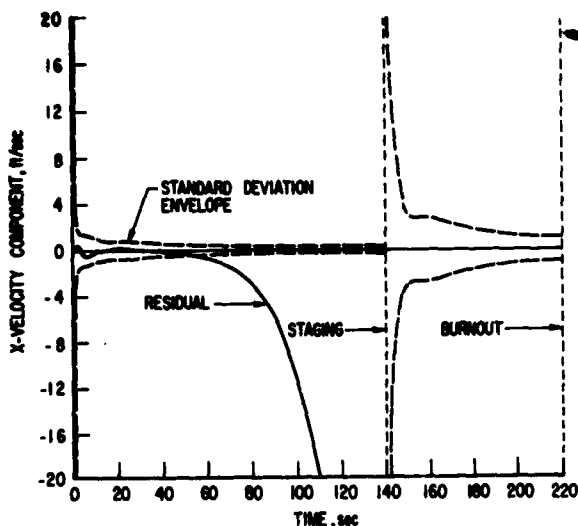
results of simulation g in which c* was not estimated but the c* errors accounted for. The position, velocity, and attitude estimate residuals are nearly the same in both simulations. Note that the filter position gains of Fig. 9b are always greater than those of Fig. 7 during the first stage demonstrating that the inclusion of the c* statistics in the filter results in greater weighting of the measurements than when they are ignored. The gains of Fig. 9b also lie above those in Fig. 9a. Since the c* estimate error decreased with time in the simulation whose results are shown in Fig. 9b, they therefore had a smaller effect when the state estimate was propagated. This error decrease is implicitly accounted for in the filter equations in the covariance matrix computations.

Fig. o. Velocity Residual; White Noise Filter, c* not Estimated

Fig. 8a. Velocity Residual, White Noise Filter
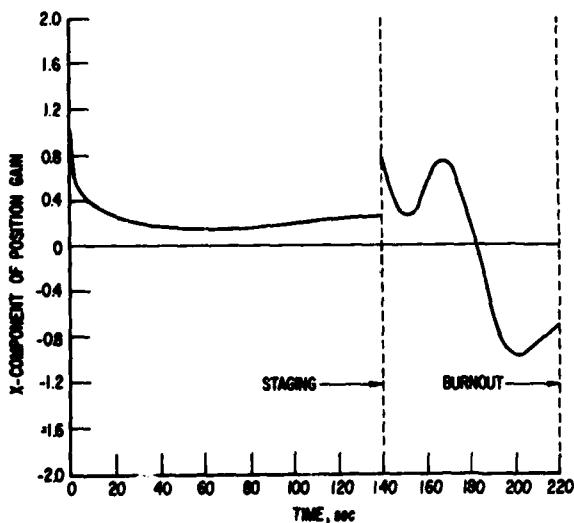

Fig. 7. Filter Gain, White Noise Filter, c* not Solved for, c* Errors Ignored
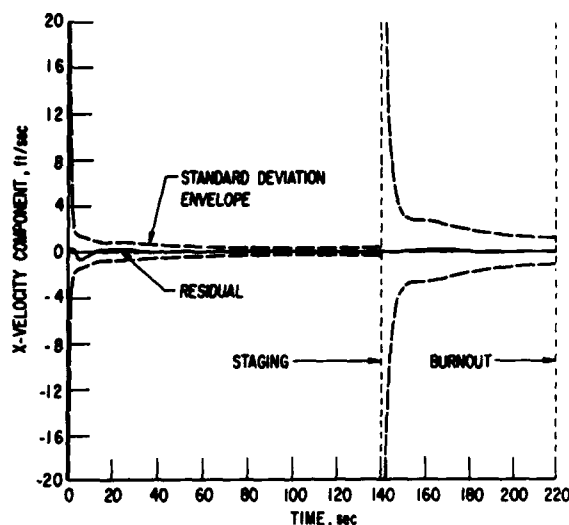
Figure 8a shows the results of simulation f in which c* was estimated, and Fig. 8b shows the
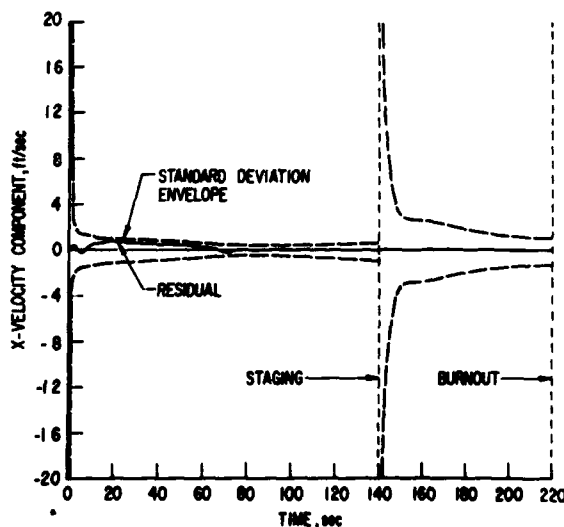

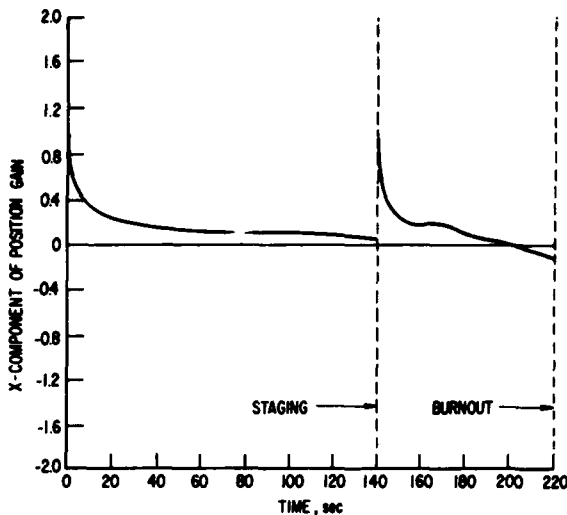Fig. 8b. Velocity Residual, c* Errors Accounted for

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Fig. 9a. Filter Gain, White Noise
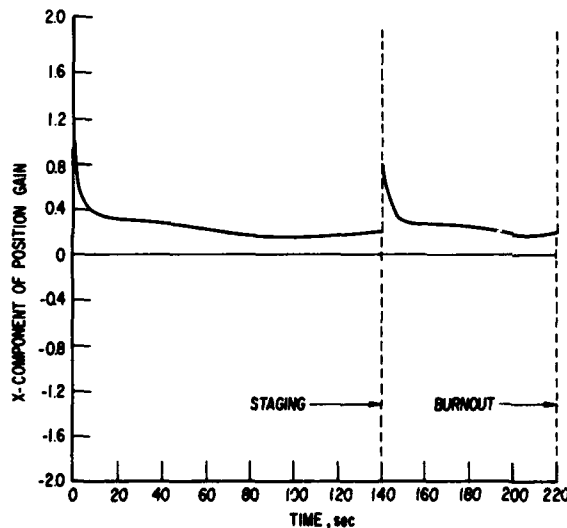Filter, c* Solved for

Fig. 9b. Filter Gain, White Noise Filter,
c* not Solved for, c* Errors
Accounted for

VI. Conclusions

The simulation results indicate, as would be expected, that the colored noise filter performance in a colored noise environment is superior to the performance of a white noise filter in the same noise environment since the colored noise filter estimation errors are smaller. However, the degree of improvement is minimal, and in a practical sense, the improvement would not warrant the increased complexity of the guidance equations. Similarly, the bias filter does not, for this application, exhibit enough performance improvement to justify its implementation.

State vector estimates which do not contain c* as one of its elements can be satisfactorily obtained if the c* error statistics are properly accounted for in the filter equations. Some form of compensation for the omission of this parameter from the estimate is mandatory if the filter is to remain stable.

In general, the results demonstrate how caution must accompany the desire for optimality in a practical application of theoretical results. It also demonstrates how the optimal approach, when used as an analysis tool, provides useful goals or limitations upon which a practical design can be based.

Appendix
The Wiener-Hopf Equation

Define the inner product of two real valued vectors a and b to be

$$<a, b> = X_1^T E (ab^T) X_2 \qquad (A-1)$$

where a is an n-vector and b is an m-vector. Let X_1 be any real n-vector and X_2 any real m-vector such that

$$X_1 \subset X_2 \text{ for } n < m$$

$$X_1 \supset X_2 \text{ for } m < n$$

$$X_1 = X_2 \text{ for } m = n$$

An inner product of two real vectors must satisfy

a. $<\alpha X + \beta y, z> = \alpha<X, z> + \beta< y, z>$

where α and β are real scalars.

b. $< X, X> \geq 0; < X, X> = 0 \Rightarrow X = 0$

Satisfaction of condition a by Eq. (A-1) is easily demonstrated. Condition b merely states, for the inner product defined, the positive-definite property of a covariance matrix.

The vector $\delta \hat{X}_{n/n}$ of Eq. (9) in the text is the estimation error and represents the additional information required to specify the state exactly. The vector $\delta \zeta_n$ represents the information, in addition to that obtained from past data, provided by the most recent observation. If $\delta \hat{X}_{n/n}$ is an optimal estimate in a least squares sense, $\delta \hat{X}_{n/n}$ is orthogonal to $\delta \zeta_n$; i. e.,

$$<\delta \hat{X}_{n/n}, \delta \zeta_n > = 0 \qquad (A-2)$$

But if

$$X_1^T E(ab^T) X_2 = 0$$

for all X_1 and X_2, then

$$E (ab^T) = 0$$

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so that

$$E[\delta \hat{X}_{n/n} \delta \hat{\zeta}_n^T] = 0$$

$$= E[(\delta \hat{X}_{n/n-1} - K_n \delta \hat{\zeta}_n) \delta \hat{\zeta}_n^T] = 0$$

If one solves for K, the Wiener-Hopf Equation is obtained as

$$K_n = E[\delta \hat{X}_{n/n-1} \delta \hat{\zeta}_n^T] \left| E[\delta \hat{\zeta}_n \delta \hat{\zeta}_n^T] \right|^{-1}$$

Symbols and Abbreviations

$A = \left. \dfrac{\partial h}{\partial T} \right|_{\substack{X = \hat{X}_{n/n-1} \\ T = \hat{T}}} - \rho \left. \dfrac{\partial h}{\partial T} \right|_{\substack{X = \hat{X}_{n/n-1} \\ T = \hat{T}}}$

$B = E[\delta \hat{T} \delta \hat{T}^T]$

$D = \left. \dfrac{\partial \varphi}{\partial p} \right|_{\substack{X = \hat{X}_{n/n-1} \\ p = \hat{p}}}$

E = the expectation operator

$H_a = H_n \hat{\phi} - \rho H_{n-1}$

$H_n = \left. \dfrac{\partial h}{\partial X} \right|_{\substack{X = \hat{X}_{n/n-1} \\ T = \hat{T}}}$

K_n = the filter gain matrix

$L_n = E[\delta \hat{X}_{n/n} \delta \hat{T}^T]$

n = subscript associating variable with the nth time point

$n/n-1$ = subscript indicating that the variable is evaluated at the nth time point and is based upon measurements up to and including the $(n-1)^{st}$

N_n = vector of measurement noise

p = dynamic model parameter vector

$\hat{p}$ = an estimate of p

$\delta \hat{p} = p - \hat{p}$

$Q = E[\mu_n \mu_n^T]$

$R = E[W_n W_n^T]$

$S = E[\delta \hat{p} \delta \hat{p}^T]$

T = measurement model parameter vector

$\hat{T}$ = an estimate of T

$\delta \hat{T} = T - \hat{T}$

$V_n = E[\delta \hat{X}_{n/n} \delta \hat{p}^T]$

W_n = a white noise vector

X_n = the state vector

$\hat{X}_{n/i}$ = the estimate of the state vector at t_n given all measurements up to and including that at t_i

z_n = the measurement vector

ζ_n = effective measurement after differencing

$\hat{\zeta}_n$ = estimated effective measurement computed from $\hat{X}_{n/n-1}$

μ_n = white process noise vector

$\Gamma = \left. \dfrac{\partial \varphi}{\partial \mu} \right|_{\substack{X = \hat{X}_{n/n-1} \\ p = \hat{p}}}$

ρ = correlation coefficient matrix representing the correlation between N_n and N_{n-1}

$\phi = \left. \dfrac{\partial \varphi}{\partial X} \right|_{\substack{X = \hat{X}_{n/n-1} \\ p = \hat{p}}}$

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13. ABSTRACT

A set of general sequential filter equations is derived for nonlinear system dynamics and a nonlinear observation model, but is obtained with the assumption of a linear estimator. These equations, which are based upon previously developed formulas, include the colored measurement noise statistics and the statistics of nonestimated model parameter errors. Several simulations made with this filter are compared with the simulation results of an estimation procedure constructed with the utilization of the standard white noise assumptions. The difference between the white noise filter results and the colored noise filter results is found to be minimal. Instability occurred when the statistics of the effective exhaust velocity in the acceleration model were not properly accounted for.

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